EMITTANCE GROWTH IN INTENSE BEAMS*

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Abstract

Recent progress in the study of high-current, low-emittance, charged-particle beams may have a significant influence in the design of future linear accelerators and beam-transport systems for higher brightness applications. Three space-charge-induced rms-emittance-growth mechanisms are now well established: (a) charge-density redistribution, (b) kinetic-energy exchange toward equipartitioning, and (c) coherent instabilities driven by periodic focusing systems. We report the results from a numerical simulation study of emittance in a high-current radio-frequency quadrupole (RFQ) linear accelerator, and present a new semiempirical equation for the observed emittance growth, which agrees well with the emittance growth predicted from numerical simulation codes.

Introduction

The problem of obtaining high-current beams with low output-emittance is a challenging one for accelerator design. To solve this problem, we need to understand the limits on maximum beam current and minimum emittance. The current is limited by the focusing available to confine a space-charge defocused beam with finite emittance to within a given radial aperture a. Current-limit formulas have been derived for both continuous beams in periodic transport lines[1] and bunched beams in linacs[2,3] using a uniform charge-density model to calculate the space-charge force. Numerical simulation results using the computer code PARMTEQ have been shown to be in good agreement with the current-limit formulas for RFQ linacs.[2]

A deterioration of the beam quality for nonstationary initial beams as a result of rms emittance growth has been observed both in numerical simulation studies and in experimental measurements.[4] This growth can occur even when Liouville's theorem is satisfied and the true phase-space volume of the beam remains invariant.[5] A small emittance is desired not only to avoid a reduction in the beam-current limit, but also for operational reasons because of the desirability of reducing beam halo and particle loss. Furthermore, some applications place severe requirements on focusing the output beam, which can only be achieved by providing a very low emittance beam. Until recently, space-charge-induced emittance growth in rf linacs could be calculated only by computer simulation, and no analytic predictions were available to serve as guidance for high-current/low-emittance linac design, even for the ideal case of perfectly aligned beams with no nonlinear external fields and no image forces.

Space-Charge-Induced Emittance Growth in RF Linacs and Beam-Transport Systems

A new understanding of the relationship between rms emittance and space-charge field energy has led to some useful approximate equations for emittance growth. The initial suggestion for such a relationship was made[6] to explain numerical simulation results for periodic quadrupole transport. For a round continuous beam in a linear focusing channel, a differential equation relating the time rate of change of the rms emittance and field energy was derived for an arbitrary distribution as[7]

$$\frac{dE^2}{dt} = -\frac{X^2K}{2}\frac{dU_n}{dt}, \tag{1}$$

where the 4-rms emittance E is defined in terms of the second moments of displacement x and divergence x′ as $E = 4\left(\overline{x^2}\;\overline{x'^2} - \overline{xx'}^2\right)^{1/2}$. The quantiy K is the generalized perveance defined as $K = eI/2\pi\varepsilon_0 mc^3\beta^3\gamma^3$, and $X = 2\sqrt{\overline{x^2}}$ the beam radius of an equivalent uniform beam (a uniform beam, with the same current and same second moments $\overline{x^2}$, $\overline{x'^2}$, and $\overline{xx'}$ as the real beam). The dimensionless quantity U_n is called the nonlinear field energy, proportional to the difference between the space-charge field energies of the real beam and of the equivalent uniform beam. The nonlinear field energy is found to be independent of beam current and rms beam size and depends only on the shape of the charge density in real space. The U_n minimum is zero (for uniform charge density), and it increases as the charge density becomes more nonuniform. Thus, U_n is a measure of the nonuniformity of the charge density and furthermore is the field energy that is available for emittance growth.

A generalized form of Eq. (1) for a bunched beam was derived by Hofmann[8,9] and can be written for three degrees of freedom x, y, and z (with linear focusing in each plane) as

$$\frac{1}{\overline{x^2}}\frac{dE_x^2}{dt} + \frac{1}{\overline{y^2}}\frac{dE_y^2}{dt} + \frac{1}{\overline{z^2}}\frac{dE_z^2}{dt} = \frac{-32}{mc^3\beta^3\gamma^3 N}\frac{d(W-W_u)}{dt}, \tag{2}$$

where N is the number of particles in the bunch, and W and W_u are the space-charge field energies of the real beam and of the equivalent uniform beam. Equation (2) [and Eq. (1)] can be integrated for the case of an rms-matched space-charge-dominated beam with linear continuous focusing (smooth approximation for periodic focusing) because the rms beam sizes are approximately constant, independent of emittance. Equations for 4-rms emittance can be derived for both bunched and continuous nonstationary beams.[10] The result for an axially symmetric bunched beam in the transverse plane can be written

$$E_{xf}^2 = E_{xi}^2\left[\frac{2+P_i}{2+P_f}\right] - \frac{16G_x(\frac{b}{a})}{(2+P_f)}\left(\frac{K_3^2L}{\sigma_{ox}}\right)^{2/3}(U_{nf}-U_{ni}), \tag{3a}$$

and for the longitudinal plane the result is

$$E_{zf}^2 = E_{zi}^2\left[\frac{2+P_i}{2+P_f}\right]\frac{P_f}{P_i} - \frac{16P_fG_z(\frac{b}{a})}{(2+P_f)}\left(\frac{K_3^2L}{\sigma_{oz}}\right)^{2/3}(U_{nf}-U_{ni}), \tag{3b}$$

where the subscripts i and f refer to the initial and final states of the beam, a and b are the rms beam sizes in x and z, and $G_x(b/a)$ and $G_z(b/a)$ are bunch geometry factors equal to unity for a spherical bunch. These factors are given approximately as $G_x(r) = [9(1-1/3r)^2 + 2][(3r-1)/2r^5]^{1/3}/6$ and $G_z(r) = [9(1-1/3r)^2 + 2]r^{1/3}/6$, where r = b/a. The quantities σ_{ox} and σ_{oz} are zero-current phase advances of the transverse and longitudinal oscillations per focusing period L. The quantity P, called the partition parameter, is defined as $P = \overline{z'^2}/\overline{x'^2}$ and is a nonrelativistic measure of the kinetic-energy asymmetry in the rest frame of the bunch. By analogy with the continuous beam, we have defined a bunched-beam perveance in terms of the number of particles N per bunch as $K_3 = e^2N/20\sqrt{5}\pi\varepsilon_0 mc^2\beta^2\gamma^3$. For a bunched beam with current I (average over one rf cycle during the beam pulse) in a linac with rf wavelength λ, N is given by N = Iλ/ec. Two mechanisms contribute to the emittance growth in Eqs. (3a) and (3b): (1) kinetic-energy exchange between the longitudinal and transverse planes, which is zero only

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when P does not change, and (2) charge redistribution, which is zero only when U_n does not change and corresponds to an exchange between field energy and particle kinetic energy as the charge density in real space evolves. It is interesting that Eqs. (3a) and (3b) can also be derived from energy conservation in the rest frame for a space-charge-dominated beam.

Although the initial values of the parameters P and U_n are known in principle for a given initial state of the beam, we have no theory available to allow a determination of the time dependence for P and U_n. However, numerical simulation studies have shown that for highly space-charge-dominated beams in linear focusing channels, the charge density approaches a nearly uniform distribution ($U_{nf} \simeq 0$) and the beam tends to an equipartitioned state ($P_f = 1$). The time scales for these mechanisms are different; charge redistribution is very fast and can occur within about a plasma period,[7,11] whereas the slower kinetic-energy exchange process can take from a few to tens of plasma periods.[9,12] The final uniform charge density may be explained as a tendency for charge redistribution to shield the interior of the beam from the linear external force, in analogy with a cold plasma. This generally results in a matched charge-density profile consisting of a uniform central core with a Debye sheath at the beam edge, whose thickness is given by the Debye length and which becomes zero in the extreme space-charge (cold-beam) limit, resulting in a uniform charge density. Why a beam, whose interactions predominately occur through collective fields rather than collisions, should equipartition is not yet clear. Nevertheless, these assumptions about the final state of the beam can provide us with a model for predicting final emittance growth. Numerical simulation results in continuous linear focusing channels for bunched beams are in good agreement with Eqs. (3a) and (3b),[9,13] and for continuous beams with corresponding emittance growth equations.[7]

The emittance growth formulas presented above have been derived for continuous linear focusing. It is of great interest to determine whether the equations do represent a good smooth approximation to emittance growth for periodic focusing, such as is used in real linacs. In addition, an initially uniform charge density in real space ($U_{ni} = 0$) eliminates emittance growth from charge redistribution in continuous focusing systems, and it is important to determine whether this conclusion is also valid for periodic systems.

In fact, the published numerical studies do appear to support the conclusion that initially uniform beams in quadrupole periodic channels give approximately no emittance growth,[14,15] at least for $\sigma_{0x} \lesssim 60°$ to 80°. In addition, for $\sigma_{0x} \lesssim 60°$, the emittance-growth formulas for charge redistribution also seem to represent a very good approximation.[6,16] For $\sigma_{0x} \geq 90°$, significant deviations are observed from the formulas, and additional emittance growth is observed for both initially uniform and nonuniform beams.[6,14] These results appear consistent with the interpretation that for certain cases such as $\sigma_{0x} \geq 90°$ in periodic channels, another mechanism becomes important namely coherent instabilities driven by the periodic structure, which have been studied in detail for the Kapchinskii-Vladimirski (K-V) distribution.[14]

Experimental support for the validity of the charge-redistribution equation for emittance growth in periodic quadrupole beam-transport systems can be seen in the published results of the Gesellschaft für Schwerionen-forschung (GSI) experiment with an initial quasi-Gaussian beam,[17] and the Lawrence Berkeley Laboratory (LBL) experiment with an initial quasi-uniform charge density.[18] The experimentally measured emittance-growth data from the two experiments at low phase-advance (high space-charge) values are significantly different from each other, and both are in at least qualitative agreement with the prediction of the emittance-growth equation.

Study of Emittance Growth in a High-Current RFQ Linac

We have conducted a study of emittance growth in the RFQ[19,20] for a 353-MHz RFQ linac, which bunches and accelerates an initial 100-keV H^+ dc beam to 3 MeV in 262 cells of length $\beta\lambda/2$. The synchronous phase is ramped from -90° to -35° and the longitudinal- and transverse-current limits, calculated at the end of the gentle (adiabatic) buncher section are 175 mA. The zero-current phase advance σ_{0x} ranges from an initial value of 34° to a minimum value of 30° at the end of the gentle buncher, which implies that coherent instabilities are not expected to play a significant role. Numerical simulation studies were conducted with the code PARMTEQ[20] using 3600 particles per run. The design results in high transmission; the transmission value exceeds 90% at 100 mA and exceeds 82% for currents as large as 165 mA. Figure 1

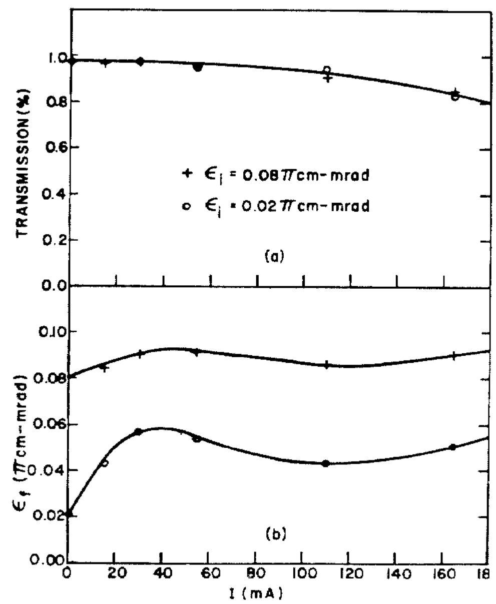

Fig. 1. Transmission and transverse 4-rms normalized final emittance versus input current for two different input emittances from numerical simultion studies of an H^+ RFQ described in the text. Smooth curves are drawn through the points.

shows the transmission and final transverse 4-rms normalized emittance (defined as $\varepsilon = E\beta\gamma$) for the transmitted beam (averaged over x and y) versus initial beam current for two different input emittances. We used an initial 4-D Waterbag distribution in transverse space (uniform filling of a 4-D hyperellipsoid volume) and a uniform filling in longitudinal position with zero energy spread. At zero beam current, there is almost no growth of emittance, consistent with the hypothesis that the observed growth at nonzero beam currents is caused by space-charge forces. For both input emittances, the final emittance rises with current to a peak near 30 to 50 mA. We see that the emittance growth is not a simple monotonic function of beam current, and is rather insensitive to beam current between about 20% and 90% of the calculated-current limit. Most of the particle losses appear to result from longitudinal effects; thus, the behavior of the transmission is nearly the same for both input emittances, and we believe that particle-loss effects are inadequate to explain the emittance curves of Fig. 1.

In Fig. 2, we show the transverse 4-rms normalized emittance versus cell number for input current 55 mA, input emittance 0.020 π·cm·mrad, and an initial Water-bag transverse distribution. Figure 2 shows that the emittance growth occurs predominately between cells 20 and 120, while the beam is being bunched. Also in Fig. 2, we show the longitudinal rms half-length $Z_{rms} = \sqrt{\overline{Z^2}}$,

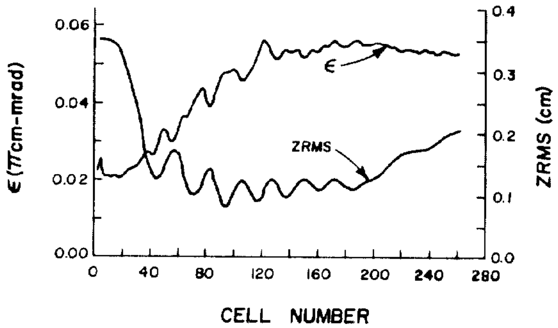

Fig. 2. Transverse 4-rms normalized emittance and rms half-bunch length versus cell number from a numerical simulation study at I = 55 mA and input emittance ε_n= 0.02 π·cm·mrad for an H^+ RFQ described in the text.

as defined in the bunch rest frame, versus cell number. The comparison of the curves, including the oscillations, suggests a strong correlation between the longitudinal compression of the beam during bunching and the growth of transverse emittance. To further support this interpretation, we show in Fig. 3 the minimum value of Z_{rms} versus beam current for seven different runs with the initial Waterbag transverse distribution and an input emittance of 0.02 π·cm·mrad. For all cases, the bunch-length minimum occurs where the transverse emittance is growing. For comparison, we show, again in Fig. 3, the corresponding final emittance curve. We see that the minimum in the bunch-length curve occurs very near the maximum in the final transverse-emittance curve. In Figs. 2 and 3, we have observed that significant emittance growth occurs while bunching the beam, with a magnitude that is related to the inverse of the bunch length. Figure 3 also suggests that the longitudinal space-charge forces are more effective in opposing the external bunching forces at higher currents (above 60 mA), and transverse space-charge forces within the bunch at 100-mA currents may actually be reduced compared to those at lower currents, resulting in less growth of transverse emittance.

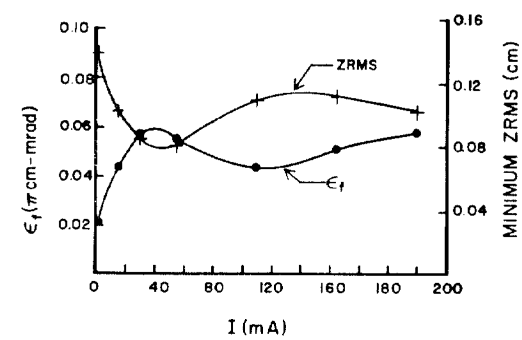

Fig. 3. Transverse 4-rms normalized final emittance and minimum value of rms bunch length within the RFQ versus input current from numerical simulation studies of an H^+ RFQ described in the text.

Figure 4 shows final transverse 4-rms normalized emittance for the transmitted beam versus input emittance for a 110-mA initial current and three different initial transverse distributions: (1) Gaussian (truncated in three standard deviations) in both position and velocity space, (2) 4-D Waterbag (parabolic charge density in position and velocity space), and (3) K-V (uniform population on the surface of a 4-D hyperellipsoid, resulting in uniform charge densities in all 2-D projections). The K-V distribution is not stationary in the RFQ because of the longitudinal fields, which are present even at the input. Figure 4 shows that the final transverse emittance is nearly the same for the three different initial distributions. Further examination shows that charge redistribution occurs very rapidly after the dc beams enter

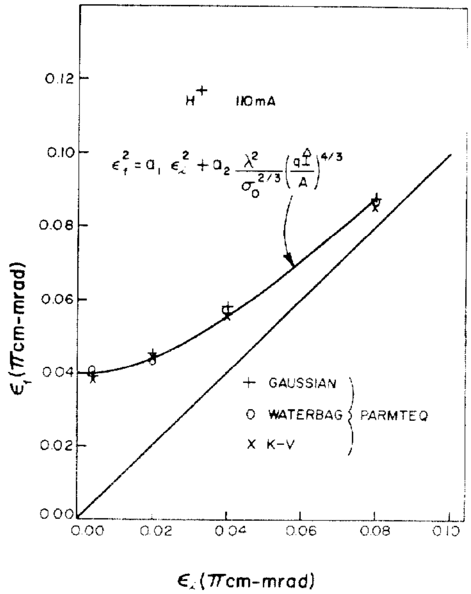

Fig. 4. Transverse 4-rms normalized final emittance versus transverse 4-rms normalized input emittance from numerical simulation studies of an H^- RFQ described in the text. Results are shown for initial Gaussian, Waterbag, and K-V distributions. The 45° line shown would correspond to no growth of emittance. The curve through the points is from the semi-empirical equation (Eq. 4).

the RFQ, resulting in a nonuniform transverse density distribution (hollow at some locations) that is nearly the same for the three different initial distributions. This charge redistribution results in a small, rapid emittance change that is largest for the initial Gaussian beam. However, after the emittance growth during the bunching, these small differences in charge distribution and transverse emittance appear to nearly vanish. In the absence of emittance growth, the results would lie along the 45° line shown in Fig. 4. Instead, the results show a lower limit on final emittance as the initial emittance decreases to zero, a phenomenon that was first reported in numerical studies of space-charge effects in drift-tube linacs.[21] The fitted curve shown in Fig. 4 is from the semiempirical equation that will be discussed in the next section.

We now summarize the main features observed in our numerical study of RFQ transverse emittance for a typical Los Alamos design: (1) the emittance growth observed in the PARMTEQ numerical simulations is predominantly caused by space-charge forces, (2) most of the growth occurs in the initial bunching section and is a strong function of the longitudinal beam size, (3) the growth is insensitive to beam current for currents in excess of about 15-20% of the current limit (an effect for which bunching may be responsible), (4) the growth is insensitive to the initial transverse distribution of the beam, and (5) the final emittance approaches a lower limit as the initial emittance approaches zero.

Emittance-Growth Model For An RFQ Linac

The emittance growth in the RFQ occurs, while many parameters (energy, accelerating field, and synchronous phase) are changing, under the influence of nonlinear external forces, especially in longitudinal space, while the beam is changing from continuous to bunched, and where particle losses can affect the results. Nevertheless, we are encouraged to begin our search for a better quantitative description of RFQ transverse-emittance growth by attempting to develop a model and a semiempirical equation based on Eq. 3(a).

We postulate that the geometry factor $G_x(b/a)$ (which increases with increased bunching, or smaller b/a) is a function of the ratio $I/\hat{I}$, where I is the current and $\hat{I}$ is the current limit. To account for the insensitivity of emittance growth to beam current for a given design, we postulate that $G_x(b/a)I^{4/3}$ is independent of $I/\hat{I}$. We propose this as an approximation, valid for $0.2 \lesssim I/\hat{I} \lesssim 0.9$. U_{nf}, and the magnitude of $G_x(b/a)$ are functions only of the detailed design procedure. Using these assumptions, Eq. 3a can be written for the 4-rms normalized emittance as

$$\varepsilon_f^2 = a_1 \varepsilon_i^2 + a_2 \frac{\lambda^2}{\sigma_{ox}^{2/3}} \left(\frac{q\hat{I}}{A} \right)^{4/3}, \tag{4}$$

where $\hat{I}$ is the (electrical) current limit in amperes (for Los Alamos RFQ designs, $\hat{I}$ represents equal limits for both the longitudinal and transverse planes), σ_{ox} in radians is the zero-current phase advance per $\beta\lambda$ at injection (after radial matching), λ is the rf wavelength in centimeters, q is the charge state, and A is the mass number in atomic mass units.

The smooth approximation formula[2] gives $\sigma_{ox} = (q/A)(eV/mc^2)(\lambda/r_0)^2/\sqrt{8}\pi$, where V is the intervane voltage and r_0 is the average radius parameter at the input (after radial matching). The coefficients a_1 and a_2 are expected to depend on the design procedure, and in particular on the longitudinal bunching and on particle losses.

In Fig. 4, Eq. (4) is shown with values $a_1 = 0.95$ and $a_2 = 1.6 \times 10^{-6}$ mrad^2/(A/amu)$^{4/3}$, which corresponds to cm·mrad units for the 4-rms normalized emittance. These choices result in a good fit to the numerical simulation results and also closely represent the results for a variety of other recent designs using the latest Los Alamos design procedures.

The second term in Eq. (4) depends on λ^2, and this strong dependence is modified only slightly in the Los Alamos design procedure when σ_{ox} (which also depends on λ) is limited by the Kilpatrick electric-breakdown criterion.[22] As part of an overall test of the scaling with respect to these parameters, we have compared the predictions of Eq. (4) with the results of PARMTEQ numerical studies using 360 particles per run of RFQ designs done several years ago for heavy-ion fusion applications.[23]

In Fig. 5, we show the final 4-rms, normalized transverse emittance versus frequency for $^{20}Ne^{+1}$ and $^{238}U^{+1}$ designs, all with injection energy 0.4 MeV and final energy 4.0 MeV. The 4-rms, normalized input emittance was 0.0132 π·cm·mrad for all designs and the input current was set equal to one-half the calculated-current limit. The current limits varied from 20 mA to 2 A, with the highest current limits at the lowest frequencies. The PARMTEQ results for ^{20}Ne are shown in Fig. 5 and compared with the results of Eq. (4), using numerical values $a_1 = 0.75$ and $a_2 = 3.2 \times 10^{-6}$ mrad^2/(A/amu)$^{4/3}$. These values of a_1 and a_2 result in a good agreement with a large variety of Los Alamos designs for the procedures used several years ago. For the ^{20}Ne results in Fig. 5, Eq. (4) compares closely with numerical calculations of the emittance growth over a range of two orders of magnitude in emittance, as the frequency varies by one order of magnitude, and the current-limit values (not shown) vary by nearly two orders of magnitude. Then the same equation gives a close prediction for the ^{238}U results, also shown in Fig. 5. Other simulation studies have been made to test the current-limit dependence of Eq. (4) at fixed values of λ and q/A and to confirm the prediction of Eq. (4) that the final emittance has no explicit dependence on β (but does depend on β through the current limit $\hat{I}$).

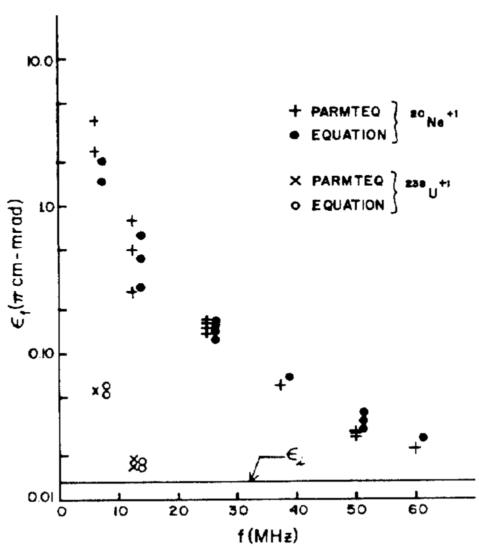

Fig. 5. Transverse 4-rms normalized final emittance versus rf frequency from PARMTEQ numerical simulation studies of RFQ designs for $^{20}Ne^{+1}$ and $^{238}U^{+1}$. The results of the PARMTEQ simulations and the semiempirical equation (Eq. 4) are shown for comparison. The input emittance for all runs is shown as a horizontal line.

Discussion

These results lead us to propose the following description of the dynamics and subsequent emittance growth in a high-current RFQ linac. The initial charge redistribution of the injected dc beam results in a rapid and relatively small exchange of field energy and transverse kinetic energy, and leads to a charge density, which is nearly independent of the injected distribution. Following this, the adiabatic bunching of the beam results in work done by the longitudinal forces, which is converted into a direct increase in space-charge field energy and longitudinal kinetic energy in the rest frame of the bunch. At each step as the beam becomes more compressed, the space-charge forces become larger and the field energy is rapidly converted to kinetic energy in both the transverse and longitudinal planes through charge redistribution. On a slower time scale, kinetic-energy exchange can occur either from longitudinal to transverse planes, which might be characteristic of a strongly bunched beam, or from the transverse to the longitudinal plane, when the bunching is weaker. Kinetic-energy exchange and charge redistribution can continue, perhaps to a lesser degree, after the bunching is completed as the energy and other parameters slowly change. The resulting emittance growth depends sensitively on the bunching process, but is nearly independent of the injected distribution. This last result is of great importance because it implies that the detailed distribution of the injected beam into an RFQ is not an important requirement for a high-brightness accelerator application.

In spite of the probable complexity suggested by this picture, we have adapted the emittance-growth equation [(Eq. 3a)] based on the two mechanisms of charge redistribution and kinetic-energy exchange as a semiempirical quantitative model. Until better experimental data are available, we will use numerical simulation results to determine the coefficients a_1 and a_2. Our description is consistent with earlier work by Jameson, who concluded that the kinetic-energy exchange mechanism was important for the RFQ.[24] Equation (4) has been of direct practical use because (1) it is allowing us to determine immediately whether or not a choice of RFQ design parameters will result in a desired final emittance, without numerical simulation computer runs; (2) it serves as input for accelerator system studies, allowing a quantitative determination of the effects on emittance of changing parameters; and (3) it is giving us direct guidance for how to proceed to improve RFQ beam-dynamics design procedures.

Equation (4) predicts that the emittance increases with increasing current limit and rf wavelength and decreases with increased transverse focusing strength (larger σ_{ox}).

The rf wavelength dependence results from the fact that at fixed current, the number of particles per bunch increases in proportion to λ. The lack of explicit dependence on β implies that at fixed-current limit, the emittance growth is independent of RFQ injection energy. The existence of a minimum final emittance at small input-emittance values is a consequence of a transfer of field energy to kinetic energy as the beam bunching increases the space-charge force. The coefficient a_2 in Eq. (4) is expected to be a strong function of the bunching. Consequently, one expects that when the first term of Eq. (4) is small (small input emittance) compared to the second, weaker bunching forces may be necessary to avoid the growth of emittance. Likewise, when the first term of Eq. (4) is large (large input emittance), stronger bunching forces may be tolerable. The RFQ design improvements based on these ideas are currently being studied. We plan to conduct further studies to follow the evolution of the U_n and P parameters of the beam to obtain more information about the details of the emittance growth process.

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